



Universität
Zürich ^{UZH}

Heterogeneous Treatment Effect Estimation for Observational Data Using Forest-based Approaches

Guest Lecture - Causal Machine Learning

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02.02.2026

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Potential outcomes framework

Estimation approaches

Model-based forests

Comparison with causal forests

Combination of model-based forests & causal forests

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Motivation

- Postpartum hemorrhage (PPH) = Blood loss $\geq$ 500 mL within 24 hours after delivery
- Leading cause of maternal morbidity and mortality worldwide according to the WHO
- Identification of risk factors for PPH required
- Our focus is on mode of delivery:

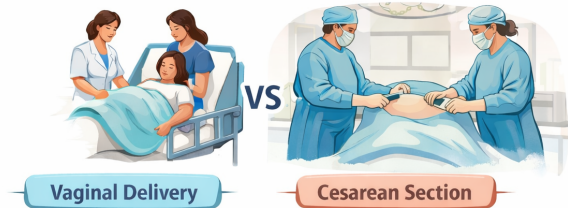
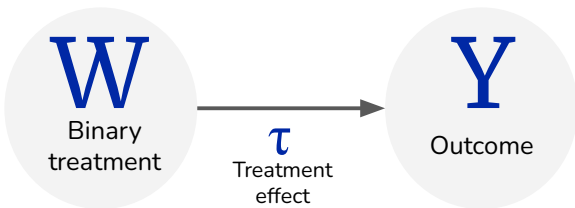


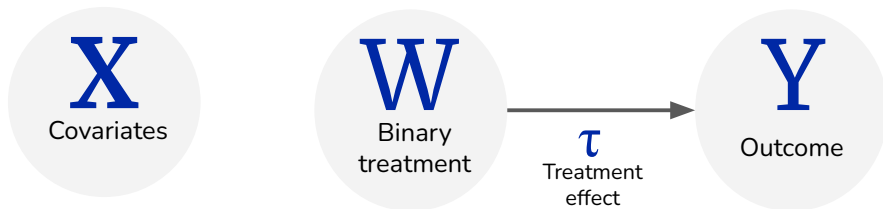
Image generated with ChatGPT (GPT 5.2)

Effect of delivery mode on blood loss



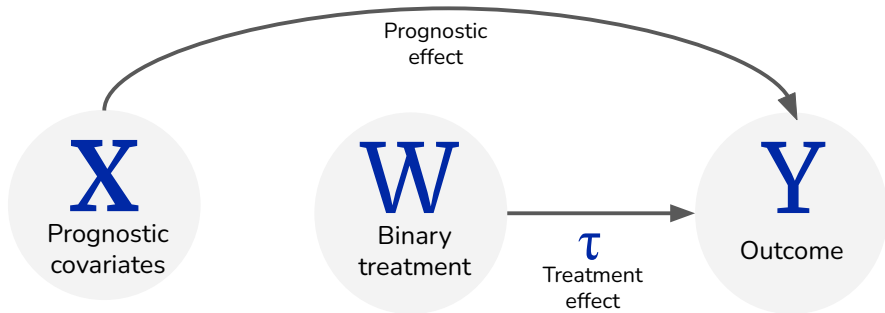
Can treatment be randomized?

Effect of delivery mode on blood loss



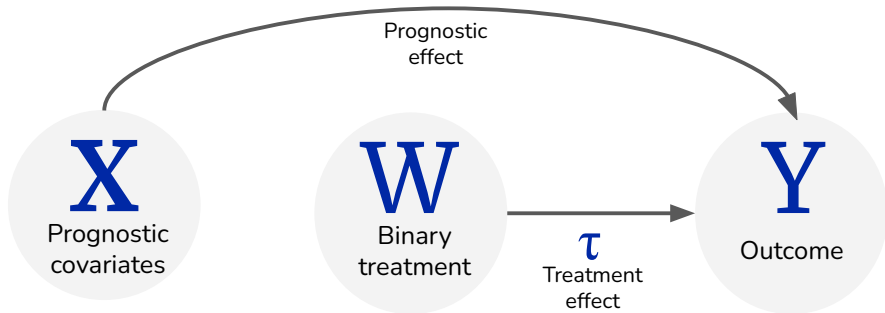
Covariates: Age, BMI, multiparity, induction of labor, etc.

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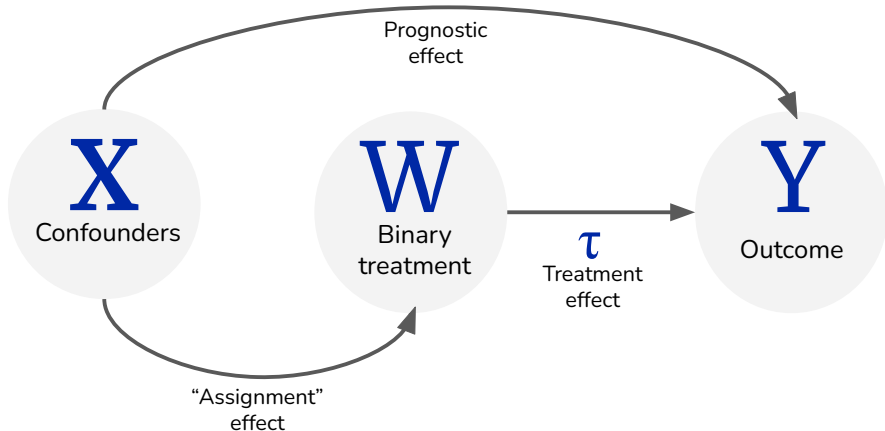
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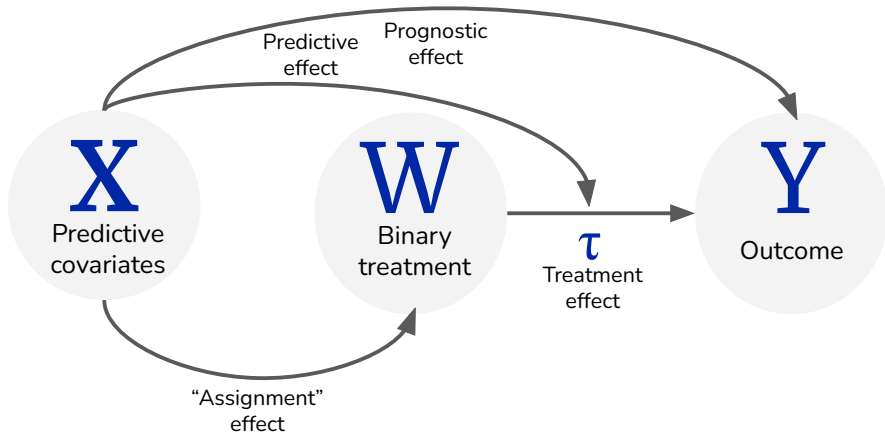
Covariates: Age, BMI, multiparity, induction of labor, etc.

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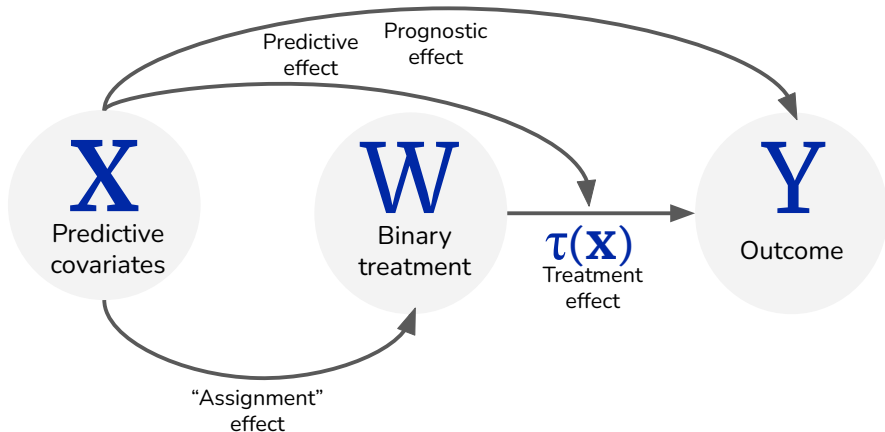
Implies that treatment effect is constant

Effect of delivery mode on blood loss



Covariates: Age, BMI, multiparity, induction of labor, etc.

Effect of delivery mode on blood loss



Covariates: Age, BMI, multiparity, induction of labor, etc.

Overview of challenges

Potential confounders

Subgroup/individual treatment effects

Association of prognostic factors and blood loss is expected to be complex, including nonlinear and interaction terms

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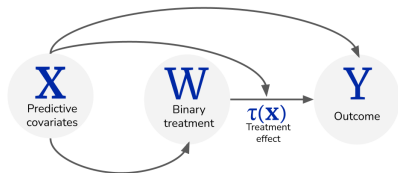
Combination of model-based forests & causal forests

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Potential outcomes framework

- Introduced by Neyman et al. (1923), extended and popularized by Rubin (1974)
- Terminology:

- Outcome $Y \in \mathbb{R}$
- (Pre-treatment) variables or covariates $\mathbf{X} = (X_1, \dots, X_p)^T$
- Treatment variable W with $W \mid \mathbf{X} = \mathbf{x} \sim \text{B}(1, \pi(\mathbf{x}))$
- $W = 1$ treatment, $W = 0$ control
- Propensity score $\pi(\mathbf{x}) = \mathbb{P}(W = 1 \mid \mathbf{X} = \mathbf{x})$
- Potential outcomes $Y(w)$, $w \in \{0, 1\}$



- Individual causal effect: $\tau := Y(1) - Y(0)$
- Fundamental problem of causal inference

Neyman et al. (1923). *Sur les Applications de la Théorie des Probabilités aux Expériences Agricoles: Essai des Principes*. Roczniki Nauk Rolniczych.
Rubin (1974). *Estimating Causal Effects of Treatments in Randomized and Nonrandomized Studies*. Journal of Educational Psychology.

Potential outcomes framework

- Approximate τ by contrasting the outcomes of similar observations in an available data set

$$D = (\mathbf{x}_i, w_i, y_i)_{i=1}^N$$

Rubin (1974). *Estimating Causal Effects of Treatments in Randomized and Nonrandomized Studies*. Journal of Educational Psychology.
Rosenbaum, Rubin (1983). *The Central Role of the Propensity Score in Observational Studies for Causal Effects*. Biometrika.
Rubin (2005). *Causal Inference Using Potential Outcomes: Design, Modeling, Decisions*. Journal of the American Statistical Association.

Potential outcomes framework

- Approximate τ by contrasting the outcomes of similar observations in an available data set

$$D = (\mathbf{x}_i, w_i, y_i)_{i=1}^N$$

- Conditional average treatment effect (CATE) for an observation $\mathbf{x}$:

$$\tau(\mathbf{x}) = \mathbb{E}(Y(1) - Y(0) | \mathbf{X} = \mathbf{x}) \stackrel{\text{A.1-4}}{=} \mathbb{E}(Y | \mathbf{X} = \mathbf{x}, W = 1) - \mathbb{E}(Y | \mathbf{X} = \mathbf{x}, W = 0)$$

- Identifying assumptions: unconfoundedness (A.1), positivity (A.2), no interference (A.3) & consistency (A.4)

Rubin (1974). *Estimating Causal Effects of Treatments in Randomized and Nonrandomized Studies*. Journal of Educational Psychology.
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Estimation approaches for $\tau(\mathbf{x})$

$$\tau(\mathbf{x}) = \mathbb{E}(Y(1) - Y(0) | \mathbf{X} = \mathbf{x}) \stackrel{\text{A.1-4}}{=} \mathbb{E}(Y | \mathbf{X} = \mathbf{x}, W = 1) - \mathbb{E}(Y | \mathbf{X} = \mathbf{x}, W = 0)$$

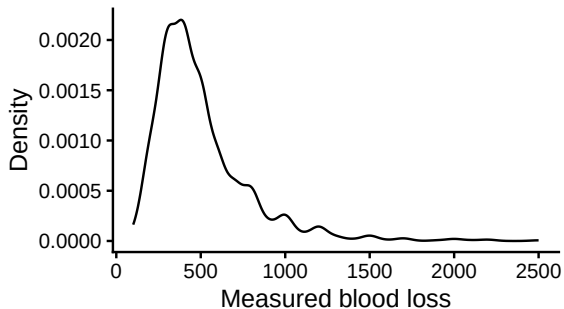
Classical approach to estimate $\mathbb{E}(Y | \mathbf{X} = \mathbf{x}, W = w)$

$$\begin{aligned} f(\mathbf{x}, w) &= \underbrace{\mu_0 + \mu_{x_1}x_1 + \dots + \mu_{x_p}x_p}_{:=\mu(\mathbf{x}) \text{ (prognostic)}} + \underbrace{(\tau_w + \tau_{x_1w}x_1 + \dots + \tau_{x_pw}x_p)}_{:=\tau(\mathbf{x}) \text{ (predictive)}} w \\ f(\mathbf{x}, w) &= \mu(\mathbf{x}) + \tau(\mathbf{x})w \end{aligned}$$

Disadvantages:

- Strong assumptions (linearity)
- Lack of flexibility in functional structures (interaction terms)

Effect of delivery mode on blood loss



High level of skewness and extreme values of blood loss measurements

ML-based approaches

- Why?
 - Weaker assumptions
 - More flexible functional structures
 - Interaction effects
- Categorization of approaches:
 1. Does the ML algorithm target $\tau(\mathbf{x})$ indirectly or directly?
 2. Can the underlying ML model be easily replaced by another ML model?

	Model-agnostic	Model-specific, e.g., random forests
Indirect	T-learner, S-learner, ...	Bivariate imputation approach, ...
Direct	Doubly robust learner, ...	Causal forest, ...

Künzel et al. (2019). *Metalearners for Estimating Heterogeneous Treatment Effects Using Machine Learning*. Proceedings of the National Academy of Sciences.
Lu et al. (2018). *Estimating Individual Treatment Effect in Observational Data Using Random Forest Methods*. Journal of Computational and Graphical Statistics.
Athey et al. (2019). *Generalized Random Forests*. The Annals of Statistics.
Kennedy (2022). *Towards Optimal Doubly Robust Estimation of Heterogeneous Causal Effects*. arXiv.2004.14497.

Overview of challenges

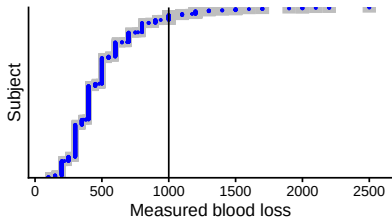
- ✓ Potential confounders
- ✓ Subgroup/individual treatment effects
- ✓ Association of prognostic factors and blood loss is expected to be complex, including nonlinear and interaction terms
- ✓ High level of skewness and extreme values of blood loss measurements

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- ✓ Potential confounders
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- ? Interval-censored outcomes to reflect measurement inaccuracy

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- ✓ Potential confounders
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Interval width of 50 mL for blood losses ≤ 1 L, 100 mL otherwise

Overview of challenges

- ✓ Potential confounders
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- ? Interval-censored outcomes to reflect measurement inaccuracy

We need a more general approach for different outcome types, censorings, etc.

Which discipline commonly works with censored outcomes?

Which discipline commonly works with censored outcomes?

⇒ Combine statistical models and machine learning approaches

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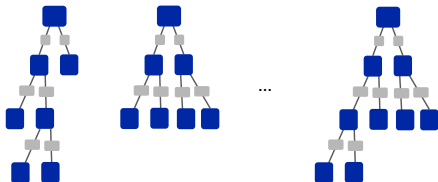
Why random forests?

Advantages:

- not much preprocessing required
- missing value handling
- easy to parallelize
- works well on data with irrelevant “noise” variables

Disadvantages:

- hard to interpret
- implementation can be memory-hungry
- prediction can be computationally demanding



Model-based forests

Aim: Estimate $\tau(\mathbf{x})$

Inputs:

Training data: $D = (\mathbf{x}_i, w_i, y_i)_{i=1}^N$

“Base” model with parameters (μ, τ) with negative log-likelihood $\ell(\mu, \tau)$

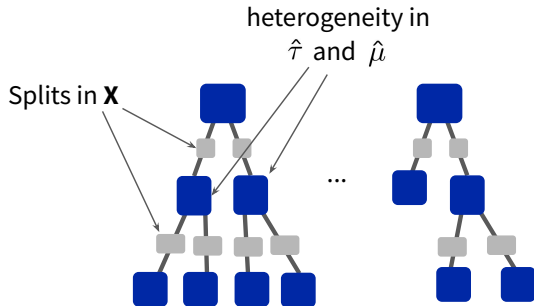
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Aim: Estimate $\tau(\mathbf{x})$

Inputs:

Training data: $D = (\mathbf{x}_i, w_i, y_i)_{i=1}^N$

“Base” model with parameters (μ, τ) with negative log-likelihood $\ell(\mu, \tau)$

In each node $\mathcal{N}$:

1. Fit $(\hat{\mu}, \hat{\tau}) = \arg \min_{\mu, \tau} \ell(\mu, \tau)$
2. Compute scores $\ell'_i(\hat{\mu}, \hat{\tau})$, for $i \in \mathcal{N}$
3. Assess association between $\ell'_i(\hat{\mu}, \hat{\tau})$ and $\mathbf{X}_i$
4. Find binary split and split data

Model-based forests - continuous outcome, linear model

Aim: Estimate $\tau(\mathbf{x})$ in $\mathbb{E}(Y \mid \mathbf{X} = \mathbf{x}, W = w) = \mu(\mathbf{x}) + \tau(\mathbf{x})w$

Inputs:

Training data: $D = (\mathbf{x}_i, w_i, y_i)_{i=1}^N$

“Base” model $\mathbb{E}(Y \mid W = w) = \mu + \tau w$ with $\ell(\mu, \tau) = \sum_{i=1}^N 1/2(Y_i - \mu - \tau w_i)^2$

In each node $\mathcal{N}$:

1. Fit $(\hat{\mu}, \hat{\tau}) = \arg \min_{\mu, \tau} \ell(\mu, \tau)$
2. Compute scores/residuals $\ell'_i(\hat{\mu}, \hat{\tau}) = (Y_i - \hat{\mu} - \hat{\tau}w)(1, w) \in \mathbb{R}^2$
3. Assess association between $\ell'_i(\hat{\mu}, \hat{\tau})$ and $\mathbf{X}_i$
4. Find binary split and split data

Model-based forests - generalized linear model

Aim: Estimate $\tau(\mathbf{x})$ in $g(\mathbb{E}(Y \mid \mathbf{X} = \mathbf{x}, W = w)) = \mu(\mathbf{x}) + \tau(\mathbf{x})w$

Inputs:

Training data: $D = (\mathbf{x}_i, w_i, y_i)_{i=1}^N$

“Base” model $g(\mathbb{E}(Y \mid W = w)) = \mu + \tau w$ with $\ell(\mu, \tau) = \sum_{i=1}^N 1/2 \ell_i(\mu, \tau)$

In each node $\mathcal{N}$:

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Example: Binary outcome, $g = \text{logit}$

$\Rightarrow$ Logistic regression base model with log-odds ratio as $\tau(\mathbf{x})$

Model-based forests - transformation model

Aim: Estimate $\tau(\mathbf{x})$ in $\mathbb{P}(Y \leq y \mid W = w, \mathbf{X} = \mathbf{x}) = F(h(y \mid \boldsymbol{\theta}) - \mu(\mathbf{x}) - \tau(\mathbf{x})w)$

Inputs:

Training data: $D = (\mathbf{x}_i, w_i, y_i)_{i=1}^N$

“Base” model $\mathbb{P}(Y \leq y \mid W = w) = F(h(y \mid \boldsymbol{\theta}) - \mu - \tau w)$ with $\ell(\mu, \tau, \boldsymbol{\theta}) = \sum_{i=1}^N 1/2 \ell_i(\mu, \tau, \boldsymbol{\theta})$

In each node $\mathcal{N}$:

1. Fit $(\hat{\mu}, \hat{\tau}, \hat{\boldsymbol{\theta}}) = \arg \min_{\mu, \tau, \boldsymbol{\theta}} \ell(\mu, \tau)$
2. Compute scores $\ell'_i(\hat{\mu}, \hat{\tau}, \hat{\boldsymbol{\theta}})$
3. Assess association between $\ell'_i(\hat{\mu}, \hat{\tau}, \hat{\boldsymbol{\theta}})$ and $\mathbf{X}_i$
4. Find binary split and split data

Effect of delivery mode on blood loss

Aim: Estimate $\tau(\mathbf{x})$ in $\mathbb{P}(Y \leq y \mid W = w, \mathbf{X} = \mathbf{x}) = \Phi(h(y \mid \boldsymbol{\theta}) - \mu(\mathbf{x}) - \tau(\mathbf{x})w)$

Inputs:

Training data: $D = (\mathbf{x}_i, w_i, y_i)_{i=1}^N$

“Base” model $\mathbb{P}(Y \leq y \mid W = w) = \Phi(h(y \mid \boldsymbol{\theta}) - \mu - \tau w)$ with $\ell(\mu, \tau, \boldsymbol{\theta}) = \sum_{i=1}^N 1/2 \ell_i(\mu, \tau, \boldsymbol{\theta})$

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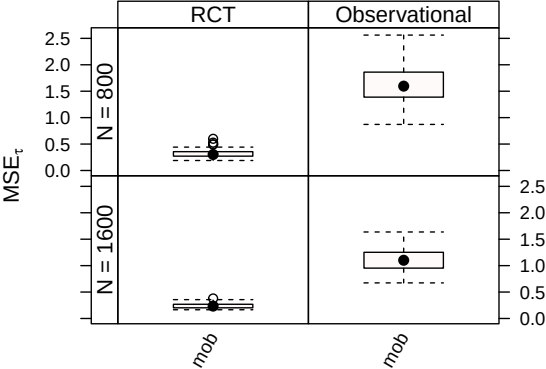
Negative log-likelihood for an interval-censored observation $(\underline{y}_i, \bar{y}_i)$

$$\begin{aligned} \ell_i(\mu, \tau, \boldsymbol{\theta}) = -\log(\mathbb{P}(\underline{y}_i < Y \leq \bar{y}_i \mid W = w)) = & -\log(\Phi(h(\bar{y}_i \mid \boldsymbol{\theta}) - \mu - \tau w) \\ & - \Phi(h(\underline{y}_i \mid \boldsymbol{\theta}) - \mu - \tau w)) \end{aligned}$$

Can we accurately estimate $\tau(\mathbf{x})$
in case of **observational data**
with model-based forests?

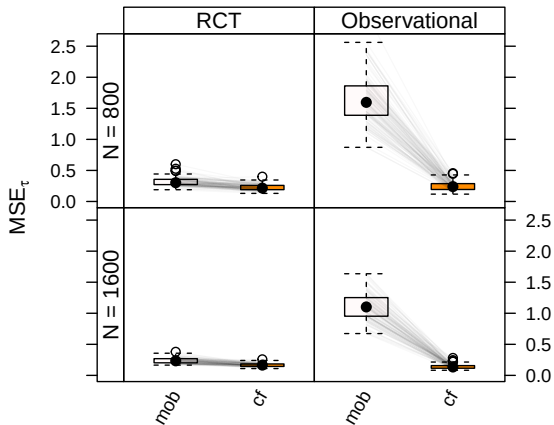
Simulation study - continuous outcome

Model-based forests (mob)



Simulation study - continuous outcome

Model-based forests (mob) vs.
causal forests (cf) by [Athey et al.](#)
(2019)



Overview of challenges

- X Potential confounders
- ✓ Subgroup/individual treatment effects
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Model-Based Forests (MOB)

- by Seibold, Zeileis and Hothorn (2018)

Causal Forests (CF)

- by Athey, Tibshirani and Wager (2019)

Seibold et al. (2018). *Individual Treatment Effect Prediction for Amyotrophic Lateral Sclerosis Patients*. Statistical Methods in Medical Research.
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- general framework for diverse outcomes

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⇒ Impression of rather distinct methodologies

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- applicability to observational studies
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Insight 1: Same theoretical basis

Model-based forests:

$$\begin{aligned}(Y \mid \mathbf{X} = \mathbf{x}, W = w) &= \mu(\mathbf{x}) + \tau(\mathbf{x})w + \sigma\epsilon \quad \text{with } \sigma > 0, \mathbb{E}(\epsilon \mid \mathbf{X}, W) = 0 \\ \mathbb{E}(Y \mid \mathbf{X} = \mathbf{x}) &= \mu(\mathbf{x}) + \tau(\mathbf{x})\pi(\mathbf{x}) := m(\mathbf{x}) \\ \text{with } \pi(\mathbf{x}) &= \mathbb{P}(W = 1 \mid \mathbf{X} = \mathbf{x})\end{aligned}$$

Insight 1: Same theoretical basis

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Causal forests:

$$\begin{aligned}(Y \mid \mathbf{X} = \mathbf{x}, W = w) &= \boxed{m(\mathbf{x}) - m(\mathbf{x})} + \mu(\mathbf{x}) + \tau(\mathbf{x})w + \sigma\epsilon \\ &= \dots \\ &= m(\mathbf{x}) + \tau(\mathbf{x})(w - \pi(\mathbf{x})) + \sigma\epsilon\end{aligned}$$

Insight 2: Simultaneous estimation vs. 2-step approach

Model-Based Forests

$$Y = \mu(\mathbf{x}) + \tau(\mathbf{x})W + \sigma\epsilon$$

Causal Forests

$$Y = m(\mathbf{x}) + \tau(\mathbf{x})(W - \pi(\mathbf{x})) + \sigma\epsilon$$

Insight 2: Simultaneous estimation vs. 2-step approach

Model-Based Forests

Estimate $\mu(\mathbf{x})$ and $\tau(\mathbf{x})$ by minimizing:

$$\frac{1}{2} (Y - \mu(\mathbf{x}) - \tau(\mathbf{x})w)^2$$

Causal Forests

Estimate $\tau(\mathbf{x})$ by minimizing:

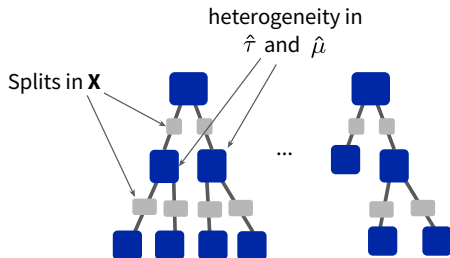
$$\frac{1}{2} (Y - \hat{m}(\mathbf{x}) - \tau(\mathbf{x})(w - \hat{\pi}(\mathbf{x})))^2$$

Insight 2: Simultaneous estimation vs. 2-step approach

Model-Based Forests

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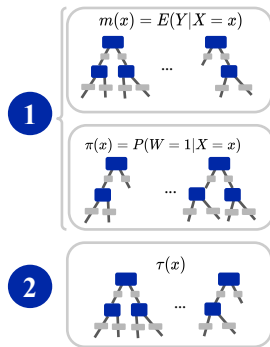
$$\frac{1}{2} (Y - \mu(\mathbf{x}) - \tau(\mathbf{x})w)^2$$



Causal Forests

Estimate $\tau(\mathbf{x})$ by minimizing:

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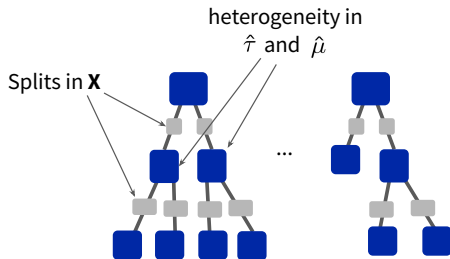


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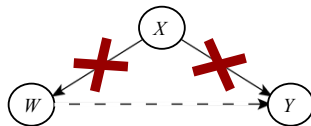


Causal Forests

Estimate $\tau(\mathbf{x})$ by minimizing:

$$\frac{1}{2} (Y - \hat{m}(\mathbf{x}) - \tau(\mathbf{x})(w - \hat{\pi}(\mathbf{x})))^2$$
$$\frac{1}{2} (\hat{Y} - \tau(\mathbf{x})(\hat{w}))^2$$

Local centering (Robinson (1988)):
Robustness to confounding



Insight 3: Similar score-based splitting criteria

Model-Based Forests

Estimate $\mu(\mathbf{x})$ and $\tau(\mathbf{x})$ by minimizing:

$$\frac{1}{2} (Y - \mu(\mathbf{x}) - \tau(\mathbf{x})w)^2$$

Causal Forests

Estimate $\tau(\mathbf{x})$ by minimizing:

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Insight 3: Similar score-based splitting criteria

Model-Based Forests

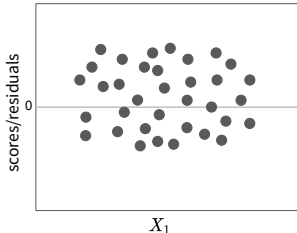
Estimate $\mu(\mathbf{x})$ and $\tau(\mathbf{x})$ by minimizing:

$$\frac{1}{2} (Y - \mu(\mathbf{x}) - \tau(\mathbf{x})w)^2$$

In each node $\mathcal{N}$

1. Derive scores $\forall i : (\mathbf{x}_i, w_i, y_i) \in \mathcal{N}$

$$s^{(i)}(\hat{\mu}, \hat{\tau}) = (y_i - \hat{\mu} - \hat{\tau}w_i)(1, w_i)^T$$



Causal Forests

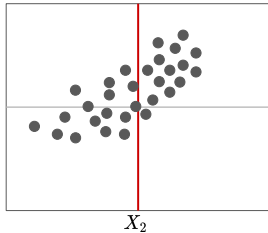
Estimate $\tau(\mathbf{x})$ by minimizing:

$$\frac{1}{2} (\hat{Y} - \tau(\mathbf{x})\hat{w})^2$$

In each node $\mathcal{N}$

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$$s^{(i)}(\hat{\tau}) = (\hat{y}_i - \hat{\tau}\hat{w}_i)\hat{w}_i$$



Insight 3: Similar score-based splitting criteria

Model-Based Forests

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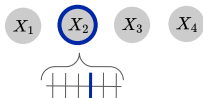
$$\frac{1}{2} (Y - \mu(\mathbf{x}) - \tau(\mathbf{x})w)^2$$

In each node $\mathcal{N}$

1. Derive scores $\forall i : (\mathbf{x}_i, w_i, y_i) \in \mathcal{N}$

$$s^{(i)}(\hat{\mu}, \hat{\tau}) = (y_i - \hat{\mu} - \hat{\tau}w_i)(1, w_i)^T$$

2. Conduct unbiased recursive partitioning using permutation tests based on $s(\hat{\mu}, \hat{\tau})$



Causal Forests

Estimate $\tau(\mathbf{x})$ by minimizing:

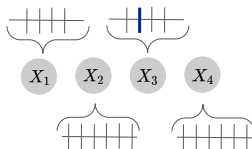
$$\frac{1}{2} (\hat{Y} - \tau(\mathbf{x})\hat{w})^2$$

In each node $\mathcal{N}$

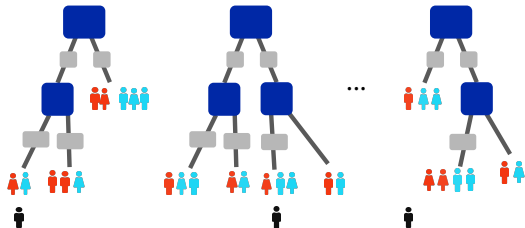
1. Derive scores $\forall i : (\mathbf{x}_i, \hat{w}_i, \hat{y}_i) \in \mathcal{N}$

$$s^{(i)}(\hat{\tau}) = (\hat{y}_i - \hat{\tau}\hat{w}_i)\hat{w}_i$$

2. Conduct (efficient) CART splitting with pseudo-outcomes based on $s(\hat{\tau})$



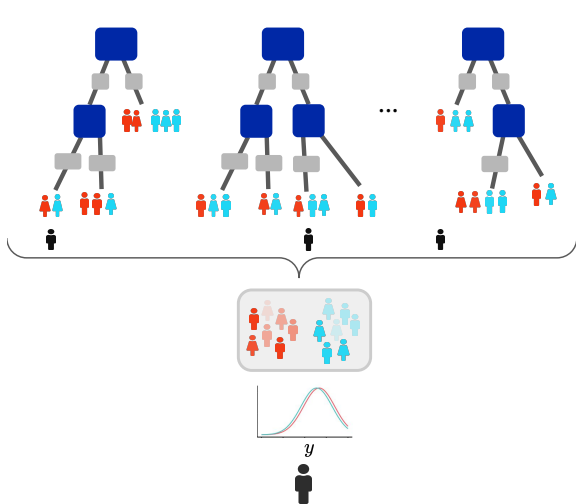
Insight 4: Same aggregation



Approach to derive $\tau(\mathbf{x})$ for a new $\mathbf{x}$ :

1. Compute frequency with which each training sample falls in the same node as 

Insight 4: Same aggregation

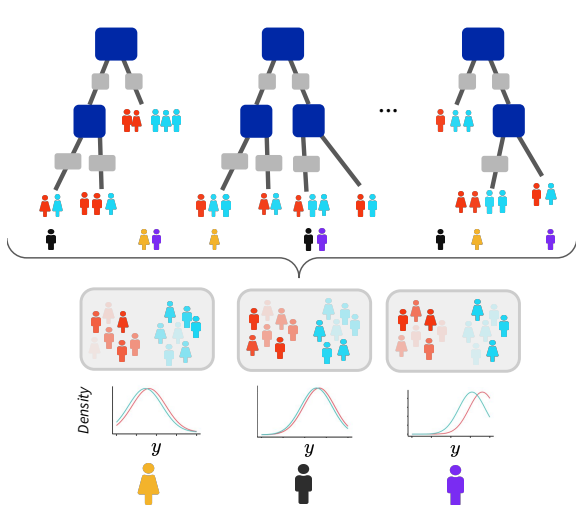


Approach to derive $\tau(\mathbf{x})$ for a new $\mathbf{x}$ :

1. Compute frequency with which each training sample falls in the same node as 
2. Estimate $\tau(\mathbf{x})$ using the reweighted dataset

⇒ Local maximum likelihood estimation

Insight 4: Same aggregation



Approach to derive $\tau(\mathbf{x})$ for a new $\mathbf{x}$ :

1. Compute frequency with which each training sample falls in the same node as 
2. Estimate $\tau(\mathbf{x})$ using the reweighted dataset

⇒ Local maximum likelihood estimation

Theoretical & methodological analysis



Foundation

Similarity 1

Theoretical basis

Difference 1

Simultaneous estimation vs. 2-step approach



Split
criterion

Similarity 2

Based on scores/residuals

Difference 2

Unbiased recursive partitioning vs. CART splitting



Prediction

Similarity 3

Estimation of $\tau(x)$ for a new x based on local maximum likelihood estimation

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Causal forest - two step approach

1. Local centering

- i. Estimate $\pi(\mathbf{x}) = \mathbb{P}(W = 1 \mid \mathbf{X} = \mathbf{x})$ and center w , i.e., $\hat{w} := w - \hat{\pi}(\mathbf{x})$
- ii. Estimate $m(\mathbf{x}) = \mathbb{E}(Y \mid \mathbf{X} = \mathbf{x})$ and center Y , i.e., $\hat{Y} := Y - \hat{m}(\mathbf{x})$

2. Estimate $\tau(\mathbf{x})$ in $\mathbb{E}(Y \mid \mathbf{X} = \mathbf{x}, W = w) = \hat{m}(\mathbf{x}) + \tau(\mathbf{x})(w - \hat{\pi}(\mathbf{x}))$ with causal forests using $\hat{Y}$ and $\hat{w}$ as input

⇒ Robustness to confounding (Robinson (1988))

Model-based forest versions

1. Local centering:

- i. Estimate $\pi(\mathbf{x}) = \mathbb{P}(W = 1 \mid \mathbf{X} = \mathbf{x})$ and center w , i.e., $\hat{w} := w - \hat{\pi}(\mathbf{x})$
- ii. Estimate $m(\mathbf{x})$ on scale of linear predictor $\mu(\mathbf{x}) + \tau(\mathbf{x})w$

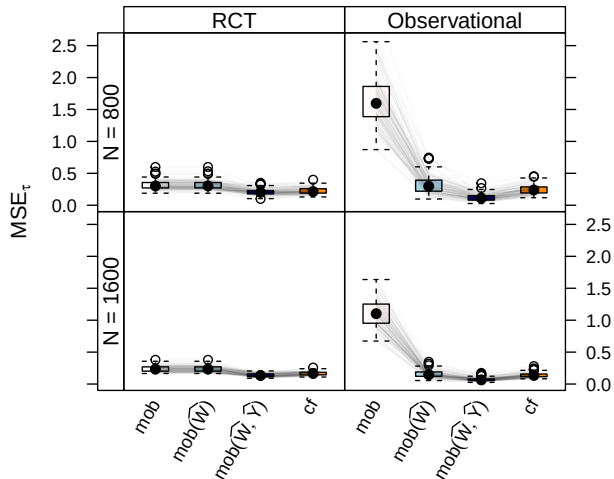
2. Estimate $\tau(\mathbf{x})$ with model-based forests by

$\text{mob}(\hat{W})$ using the centered $\hat{w}$ only

$\text{mob}(\hat{W}, \hat{Y})$ using the centered $\hat{w}$ and $m(\mathbf{x})$ as an offset in the base model

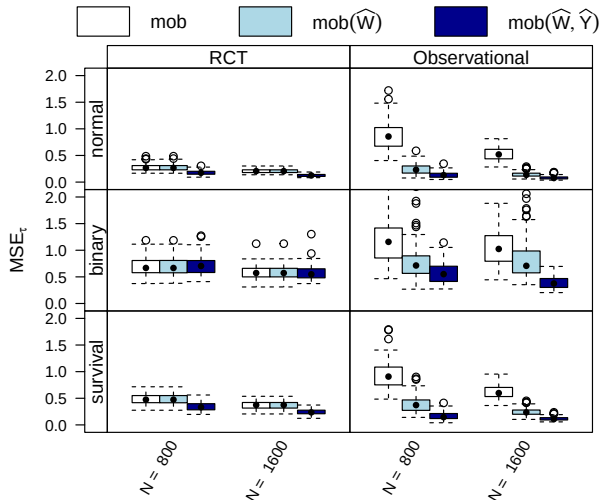
Simulation study - continuous outcome

Model-based forests (mob) vs.
mob with centered w ($\text{mob}(\hat{W})$),
mob with intercept m ($\text{mob}(\hat{W}, \hat{Y})$),
causal forests (cf)



Simulation study - diverse outcomes

Model-based forests (mob) vs.
mob with centered w ($\text{mob}(\hat{W})$),
mob with intercept m ($\text{mob}(\hat{W}, \hat{Y})$)



Effect of delivery mode on blood loss

- Prospective study conducted from October 2015 to November 2016 at the University Hospital Zurich
- Interval-censored outcomes
- $N = 1300, P = 8$

Description	Range
Gestational age	177–297 (days)
Maternal age	18–48 (years)
Multiparity	no/yes
Body mass index	15.4–66
Multifetal pregnancy	no/yes
Neonatal weight	360–4630 (g)
Induction of labor	no/yes
Chorioamnionitis	no/yes

Effect of delivery mode on blood loss

Box-Cox type transformation model:

$$\mathbb{P}(\text{MBL} \leq y \mid \mathbf{X} = \mathbf{x}, W = w) = \Phi(h(y \mid \boldsymbol{\theta}) - \mu(\mathbf{x}) - \tau(\mathbf{x})(w - \hat{\pi}(\mathbf{x}))).$$

Interpretation of τ :

- interpreted as Cohen's d: units of the treatment effect correspond to standard deviations under the normal model
- base model $\tau = 0.823$ indicates women giving birth by cesarean section have a higher postpartum blood loss

Effect of delivery mode on blood loss

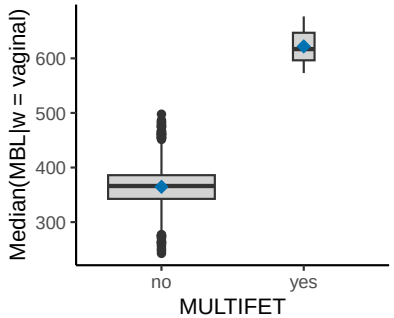
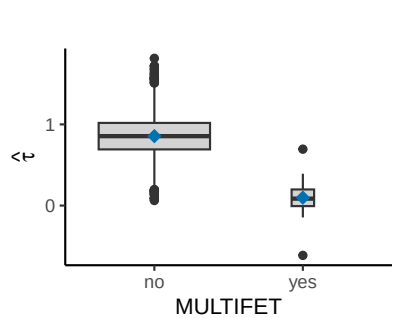
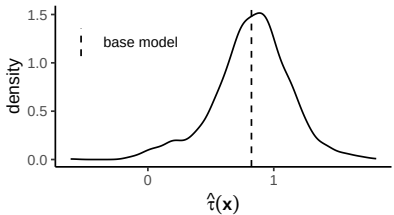
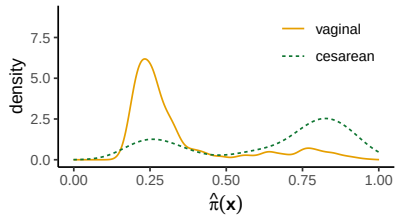


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Summary:

Versatile model framework for estimating heterogeneous treatment effects $\tau(\mathbf{x})$
in case of **observational** data and for **diverse** outcome types
based on **model-based forests** and Robinson's **orthogonalization strategy**

Conclusion

Summary:

Versatile model framework for estimating heterogeneous treatment effects $\tau(\mathbf{x})$ in case of **observational** data and for **diverse** outcome types based on **model-based forests** and Robinson's **orthogonalization strategy**

Open issues:

- Sensitivity of approaches w.r.t. violations of assumptions
- Interpretability
- Comparison to other approaches

Literature

Seibold, H., Zeileis, A., & Hothorn, T. (2016). Model-Based Recursive Partitioning for Subgroup Analyses. *The International Journal of Biostatistics*, 12(1), 45–63. DOI:10.1515/ijb-2015-0032

Seibold, H., Zeileis, A., & Hothorn, T. (2017). Individual Treatment Effect Prediction for Amyotrophic Lateral Sclerosis Patients. *Statistical Methods in Medical Research*, 27(10), 3104–3125. DOI:10.1177/0962280217693034

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